PROPAGATION OF A JET ALONG A POROUS WALL

K. E. Dzhaugashtin

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A numerical solution of the problem of self-similar development of a jet of viscous incompressible fluid moving along a porous wall is obtained within the framework of laminar boundary layer theory.

The problem of the propagation of a laminar semi-bounded jet of incompressible fluid along a porous wall has been solved numerically on the assumption that the variation of the velocity components at the wall satisfies the condition of self-similar development of the jet. Similar problems were solved in [1] and [2]. In [1] the solution was obtained for a constant value of the transverse velocity component at the wall. In [2] the problem was solved for a power law of variation of the transverse velocity component along the wall, but the relation between the self-similar solutions obtained and the actual velocity field remained undetermined.

Our object has been to study the effect of the suction or injection velocity on the attenuation of the jet and to determine the friction stresses at the wall and the location of the velocity maximum. The numerical calculations were carried out on an EMU-10 analog computer.

The starting equations of the problem are the laminar boundary layer equations

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial u}{\partial y^2},\qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

with the boundary conditions

$$u=u_w,\quad v=v_w\quad \text{при}\quad y=0,$$
$$u=0\quad \text{при}\quad y=+\infty, \tag{2}$$

where the subscript w denotes values of the velocity components at the wall. We find the solution of Eqs. (1) in the following form:

$$u=u_m F'\,\varphi,\quad u_m=A_x^{\alpha},\quad \varphi=Bx^{\frac{\alpha-1}{2}}y. \tag{3}$$

These relations also determine the law of variation of the velocity components at the wall:

$$u_w=A_x^{\alpha}F^1(0);\; v_w=-\frac{A}{B}x^{\frac{\alpha-1}{2}}\frac{\alpha+1}{2}F(0). \tag{4}$$

After transformations of the variables in the starting equations (1), we obtain

$$F'''+2(\alpha+1)FF''-4\alpha F^2=0, \tag{5}$$

where the unknown function F must satisfy the boundary conditions

$$F=F(0),\; F'=F'(0)\ \text{at}\ \varphi=0,\; F'=0\ \text{at}\ \varphi=+\infty. \tag{6}$$

The jet attenuation constant α was determined by integrating Eq. (5). For this purpose we assigned values of F(0) and F'(0) at the wall. We then varied the value of F"(0) so that the boundary conditions at infinity $F'_{(\infty)}=0$ and the equation $F'_{max}=1$ were satisfied for the given value of α.

For two particular values of α, Eq. (5) is integrable in quadratures. The value $\alpha=-0.5$ corresponds to the physically unreal case of suction from the boundary layer in the direction of motion of the fluid and, hence, is of interest from the standpoint of a check on the accuracy of the machine solution. For $\alpha=-0.5$ the solution has the form

$$F^{-1/2}F'+\frac{2}{3}\left(F^{3/2}-F^{3/2}_{(\infty)}\right)=0,$$

$$\varphi=\frac{1}{2F_{(\infty)}}\ln\left[\left(\frac{F+\sqrt{FF_{(\infty)}}+F_{(\infty)}}{F(0)+\sqrt{F(0)F_{(\infty)}}+F_{(\infty)}}\right)\times\right.$$
$$\left.\times\left(\frac{\sqrt{F_{(\infty)}}-\sqrt{F(0)}}{\sqrt{F_{(\infty)}}-\sqrt{F}}\right)^2\right]+$$
$$+\frac{\sqrt{3}}{\sqrt{F_{(\infty)}}}\left\{\operatorname{arctg}\frac{2\sqrt{F}-\sqrt{F_{(\infty)}}}{\sqrt{3F_{(\infty)}}}-\right.$$
$$\left.-\operatorname{arctg}\frac{2\sqrt{F(0)}+\sqrt{F_{(\infty)}}}{\sqrt{3F_{(\infty)}}}\right\}. \tag{7}$$

The second value, $\alpha=-1/3$, admitting a solution in quadratures, corresponds to a jet with a constant value of the momentum flow along the wall. In this case, the friction losses are compensated by the component of the momentum flow introduced with the injected mass of fluid in the direction of motion. We then have

$$F'=1-\frac{2}{3}F^2;\quad F=\frac{3}{2}+$$
$$+h\left(\sqrt{\frac{2}{3}\varphi-\operatorname{arcth}\frac{2}{3}F(0)}\right). \tag{8}$$

Comparison of solutions (7) and (8) with the machine solutions at corresponding values of the constant α shows that the maximum error in calculating the functions F and F' and the values of α, etc., does not exceed 5%.

In connection with the determination of the constants A and B, we note that in deriving Eq. (5) it was assumed that

$$A=4\nu B^2. \tag{9}$$

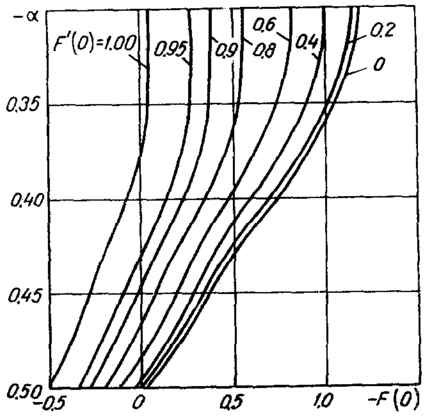

Fig. 1. Attenuation constant α as a function of injection velocity.

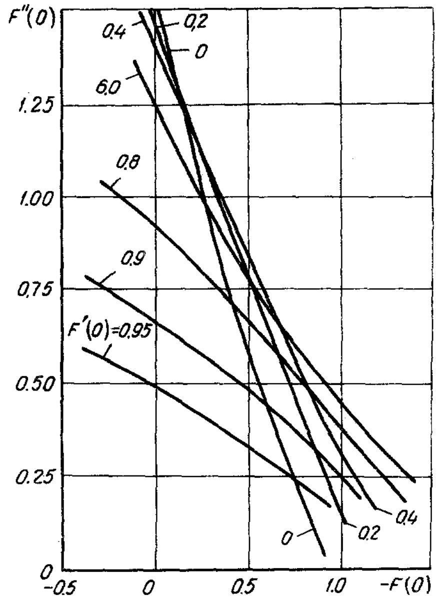

Fig. 2. Friction as a function of injection velocity.

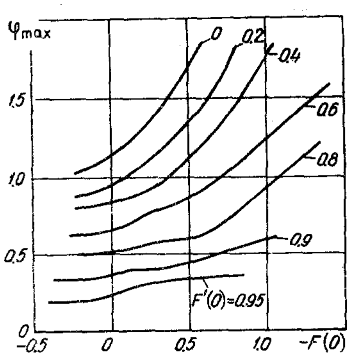

Fig. 3. Location of velocity maximum as a function of injection velocity.

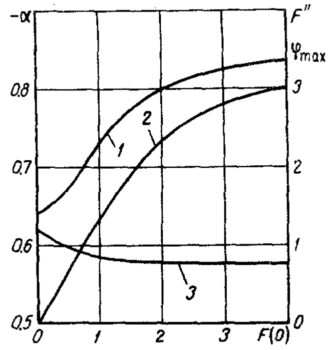

Fig. 4. Attenuation constant α (2), $F''(0)$ (1), and φ_{max} (3) as functions of the suction velocity F(0).

In the presence of power transformations of (3), an integral of the type

$$S = \int_0^\infty u^{\frac{\alpha-1}{2\alpha}}\, dy = \text{const} > 0 \tag{10}$$

does not vary along the plate [1]. Using relations (9) and (10) we determine A and B:

$$A = \left\{ \frac{\int_0^\infty [E'(\varphi)]^{\frac{\alpha-1}{2\alpha}}\, d\varphi}{2\rho\sqrt{\nu}} \right\}^{2\alpha}, \qquad B = \frac{1}{2}\sqrt{\frac{A}{\nu}}. \tag{11}$$

We proceed to discuss briefly the results presented in Figs. 1–4.

An increase in the longitudinal component of the injection velocity $F'(0)$ reduces the attenuation constant of the jet. An increase in the transverse injection velocity (proportional to F(0)) has a similar effect up to a certain point, beyond which a further slight increase in F(0) leads to a sharp increase in α (Fig. 1). It is clear from Fig. 2 that an increase in the longitudinal and transverse components of the injection velocity leads to a decrease in the shear stress at the wall:

$$\tau_w = \mu \frac{du}{dy} = 2\rho \sqrt{\frac{A}{\nu}}\, x^{\frac{\alpha+1}{2}} F''(0). \tag{12}$$

The rate of fall of $F''(0)$ with increase in F(0) is greater at lower values of $F'(0)$.

Naturally, as the normal component of the injection velocity (~F(0)) increases, the location of the velocity maximum moves away from the wall (Fig. 3). At large values of the longitudinal velocity component $F'(0)$ the change is only slight. An increase in $F'(0)$ at constant F(0) brings φ_{max} closer to the wall.

When fluid is sucked from the boundary layer, an increase in the suction rate F(0) leads to an increase in α and the friction at the wall (Fig. 4). The velocity maximum in the cross section of the jet is displaced toward the wall.

NOTATION

x, y, and z are coordinates; u and v are longitudinal and transverse velocity components; F is the dimensionless velocity profile; φ is the dimensionless coordinate; τ is the shear stress; A, B, α, β, and S are constants; ν and μ are the kinematic and dynamic viscosity; ρ is the density. Subscripts: w refers to a value of variables at the wall; m, to a maximum value.

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Senior Naval Engineering College, Leningrad